# CONNECTION BETWEEN PERIOD OF LOW-FREQUENCY COMPONENT WOLF'S NUMBERS (WNS) AND LENGTH OF WOLF'S NUMBERS SERIES 

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#### Abstract

The long-period component of Sun's activity and Gleissberg cycle are in focus of this work. The main reason is they affect daily life on Earth, for instance minimum and maximum ages change each other. For the last 60 years, many researchers have been considering that period of Gleissberg cycle limit is 80 and 110 years. Some of them consider 88 years specifically. Different researchers were analyzing WNS in different time, so they had different row length. It makes sense to analyze dependence between WNS length and period of approximation sine of low-frequency component Wolf's numbers.

Monthly Wolf number since 1749 is analyzed, the shortest part of WNS includes 18 cycles (from 1749 to 1954.37) and the longest line includes a part of a cycle No 24 (1749-2014.376). It's noticed that during increasing WNS's length, sine approximation period also increases from 84 years up to 110 years. The eighty eight years period appears at several conditions. Because of increasing period of approximation sine it's rather difficult to extrapolate sine at the future time. It should be noticed that in case of analyzing part of WNS since $1849 \div 2015$ a reliable part, period of sine approximation equals 150 years. The problem concerning matching reliable part of WNS and reconstructed part remains open.


## Introduction

Sustainable interest in long-period cycles of solar phenomena, which includes the Gleisberg cycle, is associated with the manifestation of eras of the phenomena maximum or minimum in everyday life. In the works of various authors written for the last 60 years, its period is estimated within $80 \div 110$ years. A number of researchers point out certain value of the period of the Gleisberg cycle which equals to 88 years. As various authors have analysed the Wolf numbers sequence of different lengths, it makes sense to investigate influence of length of sequence on an approximating sine of a long-period component of Wolf numbers sequence.

A minimum length of the fragment under investigation is eighteen cycles $(1749 \div 1954.37$ yy $)$; a maximum length is 24 to a cycle maximum ( $1749 \div 2014.376 \mathrm{yy}$ ). It is noted that once the sequence length increases the period of a sine increases from 84 to 110 years, and the 88 -year period comes out under certain conditions [1]. The unstable (growing) valuation of the "century" harmonic
curve period being obtained in the work complicates its extrapolation on an external time interval.

## Sine approximation

At the present day the Zurich sequence of average monthly Wolf numbers $W$ is the most representative and is widely used in various applications. Quite a complete review of these questions is presented in the monograph [2] and the review [3]. We would like to remind, that sequence of average monthly numbers $W$ includes a sequence of the regular instrumental observations since 1849 till present - a reliable sequence Wtool, and a sequence of the restored values from 1749 to 1849 - a sequence Wrest ( $\mathrm{W}=\mathrm{Wrest} \mathrm{U}$ Wtool). In the work [4] are noted the considerable characteristics differences, when comparing the properties of the sequences Wrest and Wtool. This work is based on a long-period component P1 of the sequence $W^{*}$ (a modified sequence of monthly Wolf numbers) since 1749 , which are presented on the Fig. 1.


Fig. 1. A modified sequence of monthly Wolf numbers W* and a long-period component PI

On Fig. 2 is presented the result of approximation of two options of long-period components by a sine: from 1749 to 1954.37 and from 1749 to 2014.376. The sine parameters, the period and the phase, were adjusted by the least square method, i.e. by the minimum value of a result when scanning the concerned sequences by a sine. The period was tested within $50 \div 200$ years, the phase $0 \div 2 \pi$. The studied sequences were commensurately scaled beforehand, i.e. after a subtraction of the mean value they were normalized on a root mean square. The 84 -years period is allocated for the first sequence, for the second sequence is allocated the 110-years period.


Fig. 2. Result of approximation of two options of long-period components by a sine; the axis $O X$ - sine's period.

The Fig. 3 depicts the dependence of an approximating sine period (vertical axis advanced in years) on length of a sequence; the date of the last sequence point on an axis OX is postponed. The situations of optimum approximation of a long-period component are expressly discernible by the 88-year harmonic curve. The increase of the period after the $20^{\text {th }}$ cycle, i.e. increase in proportion of a reliable data in the sequence $W$ is also notable. It speaks well to the mismatch of characteristics of the restored and certain sequences.


Fig. 3. Dependence of an approximating sine period on length of a $W^{*}$ sequence.

We would also like to note, that as the long-period component of only a reliable part of a sequence was approximated, and the period of a sine was 150 years [4].

## Conclusion

In conclusion we would like to illustrate the degree of coherence of the sequence $W$ with the 88 -year harmonic curve. Basing on the optimum
characteristics "frequency phase" for the sequence $1749 \div 1968.958$ (to the maximum of the cycle 20) the approximating sequence is comparable to $W$ numbers sequence Fig. 4. The local character of manifestation of the 88 -year harmonic curve is clearly notable.


Fig. 4. Comparing 88-year harmonic curve and Wolf numbers $W^{*}$

Gleisberg evaluated the period of the maxima envelope of solar cycles as early as in 1955 in the work [5]. The sequence $W$, by which the researchers were guided at that time, included no more than 18 cycles, a half of which refers to the restored part of the Wolf number sequence, the soundness of which calls on question. As the ratio of lengths of the Wrest and the Wtool influences the estimation value, it is logical that the different values were obtained for a "century" sequence component in works of different researchers written at different times. Range of these estimations variance conforms well to the ones received in this work.

## References

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# ЗАВИСИМОСТЬ ПЕРИОДА „ВЕКОВОЙ" ГАРМОНИКИ РЯДА ЧИСЕЛ ВОЛЬФА ОТ ДЛИНЫ ИССЛЕДУЕМОГО РЯДА 

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## Резюме

Устойчивый интерес к длиннопериодным циклам солнечной активности, в том числе к циклу Гляйсберга, связан с проявлением эпох максимума или минимума активности в повседневной жизни. В работах разных авторов, сделанных за последние 60 лет, его период оценивается в пределах $80 \div 110$ лет. Ряд исследователей выделяют конкретное значение периода цикла Гляйсберга равное 88 годам [1]. Так как разные авторы анализировали ряд чисел Вольфа различной длины, то имеет смысл исследовать влияние длины самого ряда на период аппроксимирующего синуса длиннопериодной компоненты ряда чисел Вольфа.

Минимальная длина исследуемого фрагмента - восемнадцать циклов $(1749 \div 1954.37$ гг.), максимальная длина - до максимума цикла 24 ( $1749 \div 2014.376$ гг.). Отмечено, что при увеличении длины ряда период синуса возрастает с 84 лет до 110 лет, а 88 -летний период проявляется при определенных условиях. Полученная в работе неустойчивая (растущая) оценка периода «вековой» гармоники затрудняет экстраполяцию её на внешний временной интервал.

